Questions and Answers II

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Q: we were told that $X_{\mathfrak{g}}[S^2]$ is $N = 2$ SYM, but that there was some mysterious extra data attached to S^2. What is that?

A: $X_{\mathfrak{g}}$ has local, 2d, and 4d operators. We haven't talked much about 4d operators, but this is one place where you use them. You can formulate the theory on $C \times \mathbb{R}^{3,1}$ (C a Riemann surface) and then put a 4d defect on $\mathbb{R}^{3,1}$. From the point of view of C this corresponds to marking some points in C and then assigning nilpotent orbits to them.

This is the very simplest thing to do. To get $N = 2$ SYM you need to take S^2 with four marked points and collide them in pairs. This gives S^2 with two irregular singularities.

Q: what is the notion here of allowing a singularity?

A: we should ask how the theory is modified by the presence of these singularities. The most important thing involves the moduli space of $X_{\mathfrak{g}}[C \times S^1]$ in the low-energy limit, which is the Hitchin moduli space $M_G[C]$. The singularities of various types correspond to allowing singularities of various types in the Higgs fields at the corresponding points.

Q: what is the role of the S^1 in the above?

A: recall that the moduli space of $X_{\mathfrak{g}}[C]$ itself is the base B of the Hitchin fibration. At a generic $u \in B$ this theory in the low-energy limit is a $U(1)^r$ gauge theory. It had scalar fields a^I and gauge fields $A_j^I, 0 \le j \le 3$. After compactifying on S^1 we get a new scalar field corresponding to the holonomy around the circle. We actually get two new scalar fields this way, the other coming from dual photons, and that's where the torus fibers in the Hitchin fibration come from.

Q: what is a dual photon?

A: in 3d, if A_μ is a connection, the gauge-invariant part of it is its curvature $F = dA$, which is a 2-form. Its Hodge dual $\star F$ is a 1-form, and the claim was that at least locally we could write $\star F = d\gamma$ where γ is the dual photon. This at least requires that $d \star F = 0$, which is the equation of motion for F.

(Some discussion about path integrals happened here that I didn't catch.)

(Some discussion involving Nekrasov happened here similarly.)

Q: (some question I didn't catch)

A: let's study M-theory on $T^*(C) \times \mathbb{R}^{6,1}$. Consider K different $M5$ branes on $C \times \mathbb{R}^{3,1}$ and let's talk about how to deform these. The answer involves a K-fold cover of C living in $T^*(C)$, and the space of such things is precisely the Hitchin base.

(Some discussion about M-theory constructions happened here.)

Q: what is the moduli of quantum vacua? The latest answer I've gotten is Spec of the chiral ring, but today there was a thing that wasn't affine, namely a resolution of $\mathbb{C}/\mathbb{Z}_{N+1}$.

A: we can't see the resolution because we didn't look at all operators. In our case the local operators were holomorphic functions, but there are other operators to look at.

Q: what are BPS states?

A: the Hilbert space of a SUSY theory is a representation of a Poincaré superalgebra. We'd like to describe it as a representation of this algebra. As a representation of the Poincaré algebra there are some states which are trivial; those are vacua. There are also some representations occurring discretely which we call one-particle states. In particular, inside $\mathrm{ISO}(d-1,1)$ there is a Casimir operator called the mass, and it's a meaningful question to ask what masses we get.

It's hard to compute the mass spectrum in general. But in a SUSY theory the masses are bounded below by a quantity $|Z|$ where Z is an odd symmetry called a central charge. The states of mass $|Z|$ are called BPS states.

Roughly speaking the situation is analogous to the following. The spectrum of the Laplacian on forms on a Riemannian manifold is complicated. What is not complicated is the zero eigenstates, which correspond to cohomology. In particular, these don't move when we perturb the Riemannian manifold. Similarly, BPS states are protected by some perturbations.

Wall-crossing is a subtlety that occurs when we vary some parameters, which is that the discrete and continuous states can sometimes mix. When this happens, BPS states are not protected.

Q: have the reductions of Theory X been constructed in a way mathematicians are happy with?

A [Tudor]: I would never claim to know what mathematicians are happy with.

Q: is there an abelian Theory X? Does it have a Lagrangian?

A: yes, although there are arguments about this.

Q: Witten describes the following thing in relation to abelian Theory X: take a 3-form F on a 6-manifold M and consider the Lagrangian $\int_M F \wedge \star F$, or something like that. What do we get when we do this?

A: the equations of motion would give $F = 0$. Really F should be dB for some B, and we should also incorporate the extra constraint that F is self-dual. I don't know a Lagrangian that incorporates these constraints. There are difficulties in trying to add extra fields that do this.